Calibrating predicted diameter distribution with additional information

Annika Kangas 1) and Matti Maltamo 2)

1) Researcher. Finnish Forest Research Institute, Kannus research station, P.O. Box 44, FIN-69101 Kannus, Finland. Fax. +358-6-8743201, Email: annika.kangas@metla.fi

2) Researcher. Finnish Forest Research Institute, Joensuu research station, P.O. Box 68, FIN-80101 Joensuu, Finland.[1]

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Abstract

The diameter distribution of the growing stock is an essential starting point in many forest management planning problems. There are several methods for predicting the diameter distribution of a stand, varying from methods which utilize theoretical distribution functions to non-parametric methods. Usually the predicted diameter distribution is scaled so that the stem number corresponds to the measured value. However, if stem number and basal area are both known, it may be difficult to predict a distribution that gives correct estimates for both these characteristics. Such diameter distributions can be obtained using an approach adopted from sampling theory, i.e. calibration estimation. In this study, the diameter distributions of Scots pine were predicted with two different methods, the Weibull and percentile based methods, and then calibrated with additional information. The calibration reduced the RMSE of stand variables computed from the predicted distribution.

Keywords: diameter distribution prediction, percentiles, Weibull, calibration estimation, linear programming, goal programming, nonlinear optimization

Introduction

The diameter distribution of trees is a useful tool for describing forest structure and for different forestry calculations (Loetsch et al. 1973). The stand volume characteristics are calculated using diameter distribution and treewise height and volume models (e.g. Bailey & Dell 1973). Diameter distribution is also used in predicting the growth and yield of stands in long-term management planning using treewise growth models (Siitonen 1993). Growth and yield prediction based on the diameter distribution approach has also been widely used (e.g. Clutter et al. 1983).

Different theoretical distributions, such as beta, Weibull and Johnson's SB functions, have been widely used to describe tree stock in mathematical form (e.g. Loetsch et al. 1973, Bailey & Dell 1973, Hafley & Schreuder 1977). The distribution can either be formed in terms of tree frequency or basal area (see also Gove & Patil 1998). Of these, the diameter distribution based on tree frequencies is the more widely used approach. The basal area diameter distribution has been utilized mainly in Finland (e.g. Kilkki et al. 1989, Maltamo 1998).

In applications, the diameter distribution is predicted from characteristics measured from a stand of interest. For predicting the distribution, two main methods have been applied, namely the parameter prediction method and the parameter recovery method (Hyink 1980, Hyink & Moser 1983). In the parameter prediction method, the parameters of some distribution function, for example the Weibull distribution, are predicted with regression models from measured stand characteristics. In the parameter recovery method, the parameters of the distribution function are solved from a system of equations, equating (measured or predicted) stand attributes to their analytical counterparts. The characteristics can be, for example, percentiles of diameter distribution (e.g. Bailey et al. 1981, Cao & Burkhart 1984) or moments of diameter distribution (Newby 1980, Burk & Newberry 1984). In some cases, some of the parameters are predicted and others are solved using a parameter recovery approach (Burkhart et al. 1982, Maltamo 1998).

There are also methods that do not rely on any predefined functional form. For example, Borders et al. (1987) developed the percentile based diameter distribution prediction method. This method uses a system of 12 percentiles defined across the range of observed diameters. Assuming a uniform distribution between adjacent percentiles, stem numbers in desired diameter classes can be calculated. This method has been further developed to project future stand tables (Borders & Patterson 1990, Knowe et al. 1997). Maltamo et al. (1999) used a similar approach, but smoothed the distribution function between the percentiles with a spline function.

Another possibility is to rely on purely non-parametric methods. For example, Haara et al. (1997) and Maltamo & Kangas (1998) predicted the basal area diameter distribution using nearest neighbor based approaches. The predicted diameter distribution was a weighted average of the distributions of accurately measured stands most similar to the stand of interest.

The predicted diameter distribution is usually scaled to the measured number of stems, so the stem number obtained from the distribution corresponds to the known characteristic. With respect to other characteristics measured from the stand, the situation is more complicated. Using the parameter prediction method or non-parametric approach, there is no guarantee that the stand characteristics obtained from the predicted diameter distribution correspond to the measured stand characteristics. The same holds for the percentile based method, if all percentiles are predicted with models. With the parameter recovery method, the compatibility of predicted and measured stand characteristics can be guaranteed for the characteristics used for solving the parameters of the diameter distribution function. However, if the characteristics used in parameter recovery are predicted using models (e.g. Burk &

Newberry 1984), the compatibility of the original stand characteristics and values obtained from the diameter distribution cannot be guaranteed.

If both basal area and number of stems are measured, the compatibility between the measured values and values obtained from the distribution is important. Using a distribution which provides incorrect values for known characteristics means wasting information.

In this study, diameter distributions of Scots pine are calibrated in order to produce a distribution which is compatible with all information available from the stand. The aim is to examine how the calibration affects the accuracy of the predicted distribution in different situations. The method is based on the calibration estimation method of Deville and Särndal (1992). It can be applied to any distribution function, obtained with any method available. The methods used in this study for predicting the diameter distribution are the three-parameter Weibull distribution and the percentile based method.

Material and methods

Material

The empirical data for the diameter distributions includes sample plots from 49 thinned and 49 unthinned Scots pine (*Pinus sylvestris* L.) stands. Both data categories include young and middle-aged (40–80 yrs) stands on sub-xeric and mesic heaths. Diameter at breast height was recorded from all the Scots pines included in the sample plot.

The thinned stands consist of the permanent INKA sample plots from eastern Finland established by the Finnish Forest Research Institute (Gustavsen et al. 1988). INKA sample plots consist of three separate circular sub-plots (Gustavsen et al. 1988), which in this study were combined in the calculations. The area of the combined sample plots of thinned stands varied from 339 m^2 to 0.21 hectares.

The unthinned stands included data collected from the Republic of Karelia and the Leningrad district. These stands can be considered to be in a natural state. The sample plot network was originally established by the Finnish Forest Research Institute, the Faculty of Forestry at the University of Joensuu and the Forest Engineering Faculty at the University of Petrozavodsk. The data consisted of square-shaped sample plots of 0.1 hectares in size. In addition, from the study of the state of Karelian forests, five sample plots located on the Karelian Isthmus were included in the unthinned data. Each of the sample plots in Karelian Isthmus was a circular plot of 300 m^2.

Diameter distribution models

Two different methods were applied to predict the diameter distribution, namely the method based on the Weibull function and the method based on percentiles. It was assumed that the number of stems in the stand is known and the estimated relative frequencies in the diameter classes are scaled to the correct stem number. The diameter classes applied were 1 cm wide. The estimate of the basal area of a stand is calculated from the diameters and frequencies of the predicted distribution and thus it includes an estimation error.

The first method applied was the percentile based diameter distribution prediction method (Borders et al. 1987, Maltamo et al. 1999). In this method, logarithms of 13 percentile points (0, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95 and 100 %) were used to estimate the diameter distribution. The models for predicting these percentiles were estimated from this same data by Maltamo et al. (1999).

The predictors used in the models were the median diameter, d_{med}, the basal area median diameter, d_{gM}, and a dummy variable describing whether the stand is thinned. The value of the cumulative diameter distribution F for each diameter class was obtained by interpolating between the predicted values of percentiles with Späth's rational spline interpolation (Späth 1974, Lether 1984, see Maltamo et al. 1999). Subsequently, the relative frequency in a diameter class $[a, b]$ could be calculated as $F(b)$-F(a).

Another method used to predict the diameter distribution was the three-parametric approach of the Weibull function (e.g. Bailey & Dell 1973) . The probability density function of the three-parametric Weibull distribution for a random variable x is

$$f(x)=\begin{cases}\frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1}\exp\left(-\left(\frac{x-a}{b}\right)^{c}\right), & (a \le x < \infty)\\ 0, & (x<a)\end{cases} \qquad (1)$$

where a is the location parameter, b is the scale parameter and c is the shape parameter. Parameters a and c were predicted using the models of Maltamo et al. (1999). Parameter b was solved from the cumulative distribution function of the Weibull distribution, with $d_{med} = F(0.5)$ as

$$b=\frac{d_{med}-a}{(-\ln(0.5))^{\frac{1}{c}}} \quad (2)$$

The median diameter, the basal area median diameter, the basal area, the number of stems and a dummy variable describing whether the stand is thinned were used as predictors in the models for the Weibull parameters.

Calibrating the predicted diameter distribution

The calibration of diameter distribution was performed with an approach similar to the one used by Deville and Särndal (1992) in survey sampling. In their study, a starting point for the calibration was the common Horvitz-Thompson estimator for estimating the total of y in a population, $\hat{t}_{y\pi}$

$$\hat{t}_{y\pi} = \sum_s y_k / \pi_k \quad (3)$$

where s is the sample, $k \in s$, and π_k is the inclusion probability due to sampling design. The basic sampling weights $q_k = 1/\pi_k$ were, however, modified with known totals t_x of auxiliary variable x to form the calibration estimator

$$\hat{t}_{yw} = \sum_s w_k y_k \quad (4)$$

respecting the calibration equation

$$\sum_s w_k x_k = t_x . \quad (5)$$

The calibration is used in order to obtain a perfect estimate for each auxiliary variable available from the sample at hand. The calibrated weights w_k are chosen so that they are as close as possible to the original weights q_k. The modified weights are estimated by minimizing some distance measure, for example a quadratic distance (Deville & Särndal 1992)

$$\sum_s (w_k - q_k)^2 / q_k , \qquad (6)$$

subject to the calibrating equation. This distance measure leads to a regression estimator. Deville & Särndal (1992) present several alternative distance measures, which lead to a family of calibration estimators. Some of the distance measures may produce negative or extreme values, which may be unacceptable. With some distance measures, solutions to the problem may not exist. Consequently, Deville & Särndal (1992) propose that numerical features and ease of computation form the basis for choosing the distance measure.

In this study, the calibration estimator was used to modify the predicted frequencies f_k of each diameter class $k = 1, ..., K$, where K is the number of diameter classes. The modification was done so that the modified frequencies w_k are as close as possible to the predicted frequencies f_k, while respecting the calibration equation(s). The observed basal area was used as the known total. The calibration equation used was

$$\sum_{k=1}^{K} w_k g_k = G, \quad (7)$$

where G is the stand basal area (m^2/ha) and g_k is the treewise basal area in class k (m^2/ha), obtained using the mid-point diameter in that class. Further, it was required that the total stem number, which was used in scaling the original frequencies, remained correct also after the calibration, and this was ensured by using the calibration equation

$$\sum_{k=1}^{K} w_k = N \qquad (8)$$

where N is the stem number per hectare. Another characteristic used in the calibration was the basal area median diameter. In the calibration d_{gM} was used by setting the basal area below the mid-point diameter of the corresponding diameter class to half of the total basal area

$$\sum_{k=1}^{K} w_k \gamma_k = G/2, \quad (9)$$

where

$$\gamma_k = \begin{cases} g_k,\ d_k < d_{gM} \\ g_k / 2,\ d_k = d_{gM} \\ 0,\ d_k > d_{gM} \end{cases}.$$

The sum of the absolute deviations from the original frequencies

$$\sum_{k=1}^{K} |w_k - f_k| \qquad (10)$$

was used as one distance measure. This distance measure allows the use of linear optimization. This distance measure was minimized subject to the calibration equation(s) with a revised simplex algorithm (IMSL subroutine). The problem was formulated using a goal programming approach as (e.g. Taha 1997)

$$\min \sum_{k=1}^{K} \left(s_k^- + s_k^+ \right) \qquad (11)$$

subject to

$$w_k - s_k^- + s_k^+ = f_k, \ k = 1, \dots, K$$

$$\sum_{k=1}^{K} w_k g_k = G$$

$$\sum_{k=1}^{K} w_k = N$$

$$\sum_{k=1}^{K} w_k \gamma_k = G/2$$

$$s_k^-, s_k^+, w_k \geq 0, \forall\, k .$$

Other distance measures tested were the quadratic distance (eq. 6), a logarithmic distance

$$\sum_{k=1}^{K} w_k \log(w_k / f_k) - w_k + f_k , \qquad (12)$$

and a square root based distance

$$\sum_{k=1}^{K} 2(\sqrt{w_k} - \sqrt{f_k})^2 . \qquad (13)$$

Minimizing these distance measures while respecting the calibration equation(s) is a constrained nonlinear optimization problem. The problem was solved by reformulating it using Lagrange multipliers (see Deville & Särndal 1992 for details). Subsequently, a resulting group of nonlinear equations was solved using IMSL subroutines.

Comparison of the methods

The effect of calibrating the diameter distribution was examined by comparing the root mean square errors and biases of stand volume (m^3/ha), with and without calibration. The absolute root mean square error (RMSE) was calculated as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(V_i - \hat{V}_i\right)^2}{n}} \quad (14)$$

where n is the number of sample stands, V_i is the true volume of growing stock in stand *i* and $\hat{V}_i$ is the volume of stand *i* estimated from the predicted distribution. The relative RMSE of the volume estimate was calculated by dividing the absolute RMSE by the true mean volume $\overline{V}$ of the stands. The bias of the predictions was calculated as

$$bias = \frac{\sum_{i=1}^{n}\left(V_i - \hat{V}_i\right)}{n}. \qquad (15)$$

In addition to stand volume, the saw timber volume (m^3/ha), basal area (m^2/ha) and number of stems were also considered. Stand volumes were calculated with Laasasenaho's (1982) models, using only diameter at breast height as a predictor. The saw timber volume was defined as the total volume of trees larger than or equal to 16.5 cm.

Results

In this study, the diameter distributions predicted with two methods, i.e. the Weibull and percentile based methods, were calibrated with additional information. In these calculations, four distance functions and three combinations of calibration equations were used.

When the original distribution was predicted with the Weibull method, the best results with respect to stand volume were obtained using the distance function based on absolute differences (10) and by calibrating with all the information available (Table 1). The nonlinear distance functions produced nearly identical results. With the Weibull method, the predicted distributions were originally unimodal even for bimodal stands. The calibrated distribution was in some cases slightly bimodal (Fig. 1). In another example the calibration affected the kurtosis of the predicted distribution (Fig. 2).

The calibrated results, as well as the initial results, were more accurate when the percentile method was used rather than than the Weibull method (Table 2). The distance function based on absolute differences (10) produced the most accurate results and the best strategy was to use all information in the calibration. Again, the nonlinear functions produced nearly identical results. However, when using the nonlinear distance functions, the most accurate results with respect to stand volume were obtained when only the basal area was used in the calibration. Despite the calibration equations, some minor variation remained in the basal area and stem number estimates. This is most probably due to the original distributions being narrower when using the percentile method: there was not as much room for calibration.

With the percentile method, bimodal distributions could also be produced and the calibration preserved the bimodality. In some cases the calibration produced slightly bimodal distributions from unimodal predicted distributions (Fig. 3). The calibration was also able to change the skewness and mode of the predicted distribution (Fig. 4).

If the basal area of a stand calculated from a predicted distribution was overestimated, calibration removed big trees and increased the number of small trees (e.g. Fig. 5). Correspondingly, in the case of underestimation calibration increased the number of big trees and decreased the number of small trees (e.g. Fig. 4). Great differences in observed and estimated basal areas are possible, for example, in stands where there are some big trees separate from the rest of the distribution, which the predicted distribution cannot describe.

Although the RMSE% of stand volume was smallest with the distance function based on absolute differences (10), this method may not be the best. In many stands, the calibration was done by increasing a high peak to one diameter class (Fig. 5). This may not be realistic. The nonlinear distance functions all produced smoother distributions, especially when the basal area and stem number were used for calibration. Using the basal area median diameter in calibration could produce rather wiggly distributions (Fig. 6). One of the nonlinear distance functions (13) produced the smoothest distributions. Another (6) produced a few negative frequencies and is thus not desirable.

Discussion

In this paper, predicted diameter distributions were calibrated with the additional information available. The calibrated diameter distributions produced compatible estimates for the stand characteristics used in the calibration. They provided more accurate estimates of the stand volume characteristics than the uncalibrated distributions.

The study material used was partly from unmanaged stands with irregular stand structure. This could be seen in a bimodal or highly skewed form of the distribution or a high number of stems per hectare. In these kinds of stands the relationship between stem number and basal area may be complicated. This can be seen from the relatively poor accuracy of the predicted distributions. This may also have affected the calibration of the predicted distributions: the better the predicted distribution is, the easier the calibration. In plantation forests with a more regular stand structure, it would thus be easier to calibrate the diameter distribution. However, the need for calibration might also be smaller.

Of the distance functions tested, the most accurate results in terms of RMSE of stand volume were obtained with the distance function based on absolute differences (eq. 10). However, the diameter distribution produced using this function may not be realistic, because of the possible high peaks. The nonlinear distance functions produced smoother distributions. All of them produced quite similar results, except that the function based on squared differences (eq. 6) sometimes produced negative frequencies.

The effect of using calibrated diameter distributions in the growth and yield prediction was not considered. The predicted diameter distribution may, however, have a marked effect on these predictions (e.g., Maltamo & Kangas 1998). If the calibrated distribution is to be used as a basis for stand projection, the nonlinear distance functions seem to be more advisable. Confirming this, however, requires further studies.

With the percentile based method, using both stem number and basal area in the calibration produced worse results than using only the basal area. The reason for such a paradoxical result may be the fact that the percentile method produces quite narrow original distributions. Such a distribution does not give much room for calibrating.

In predicting the parameters of the Weibull function, both the basal area and stem number were assumed to be known. Yet, the results were quite inaccurate with respect to the computed stand basal area: the information available was not used efficiently. Calibrating the diameter distribution proved to be an efficient way of using all available information. If, however, all available information was efficiently used, for example in scaling the distribution or in parameter recovery, calibration might not be helpful. Improving the results would then mean measuring more information from the stands. Such additional information could include, for example, the minimum and maximum diameters of the diameter distribution.

Nevertheless, one advantage of calibration is that the same stand characteristics need not be known from each stand. Some characteristics, such as stem number and median diameter, would be needed from each stand to predict the original distribution. It would then be possible to measure additional information from some stands and to use it to produce a more accurate diameter distribution.

In Finland the prediction of diameter distribution is usually connected to compartmentwise inventory and forest planning where the information to be measured is quite scarse. Diameter distributions are predicted using probability density functions and this leads to to quite similar distributions in all kinds of forests. Although the stand characteristics are assessed by tree species and storeys, problems have arisen especially in heteregenous stands such as mixed forests, young unthinned forests, old forests and forests growing on peatlands. Calibrating the predicted distribution with the additional information available would be one solution to this problem. During the field work it may already be decided how much information is needed for each stand. In even aged and single species forests, only the essential mean characteristics are registered. If the stand is visually assessed as being heterogeneous, more information can be collected. Obviously, the most accurate information concerning the diameter distribution would be obtained from a direct sample, but this is usually too expensive for practical applications.

If the characteristics used in calibration include measurement or prediction errors, strict calibration may not be the most efficient method. In such a case, the calibration could be done with goal programming, giving different weights for the different goals (calibrating equations). Another possibility would be to use penalty functions in nonlinear programming. The penalties could be given so that those characteristics that are most accurate are given the largest penalties in calibration. The penalties could, for example, be proportional to the inverse of measurement error variance. This issue also needs to be considered in the future.

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Fig. 1. An example of the predicted and calibrated Weibull based distribution in a bimodally distributed stand.

Fig. 2. An example of the predicted and calibrated Weibull based distribution.

Fig. 3. An example of the predicted and calibrated percentile based distribution in a bimodally distributed stand.

Fig. 4. An example of the predicted and calibrated percentile based distribution.

Fig. 5. An example of the high peak with distance function (10)

Fig. 6. The effect of increasing calibration equations, distance function (13). Calibrated 1 = calibrated with basal area, Calibrated 2 = calibrated with basal area and stem number, Calibrated 3 = calibrated with basal area, stem number and diameter of basal area median tree.